Making Numbers Your Friends: A Set of "Make This Number" Activities

HELEN CHICK provides us with a variety of "Make this Number" activities that can be adapted for a range of students and contexts.

What do your students think of when you show them the number 8? Do they think of it as being the number after 7 and before 9, or have they started to build a richer picture of its relationship to other numbers? Do they see it as 1 + 7, 2 + 6, 3 + 5, and 4 + 4, and appreciate the patterns in those sums? Do they identify its factors: 1, 2, 4 and 8? Do they see it as half of 16, and recognise this as both $\frac{1}{2} \times 16$ and $16 \div 2$? Do they know that it is the square root of 64 ($\sqrt{64}$)? Do any of them see it as 2 cubed (2^3)? Would you be glad if they did?

Fluency with basic number facts is vital for students' progress in mathematics. Not only does it contribute to students' facility with mental computation and algorithms, but an understanding of numbers and their properties builds a foundation for future mathematical work including algebra.

There are many activities that can help students "make numbers their friends," so that every time students meet numbers, they know all about them, or, at least, start thinking about what they could find out about them. One of the most famous activities is "Today's number is…" (McIntosh, DeNardi, & Swan, 1994). In this activity a teacher selects a number for the day and, after allowing time for students to think, invites students to contribute properties and number sentences involving that number. Careful choice of numbers and strategic

direction of the resulting class discussion can lead to productive mathematical learning (for example, “Today’s number is 11,” might lead to a discussion of prime numbers).

A powerful aspect of “Today’s number is…” is that it turns practice with number facts backwards: it starts from the “answer,” and asks students to construct “questions.” This can be good for students who might feel pressure solving more regular “find the answer” problems, which have only one solution. There are many acceptable responses for “Today’s number is…” which means that all students can respond. Moreover, students can contribute at the limits of their current level of understanding, especially if the teacher gives them time to “think of the most difficult thing you can” (D. Neal, personal communication).

In this article I present a series of similar activities that will build students’ friendship with numbers and number properties. They are based, to some extent, on the classic number activity that used to be given at the start of the year in the 1970s and 1980s, in which students had to construct as many numbers as they could using only the digits from the current year (e.g., in 1987 you could “make” 2 as $2 = 18 - 9 - 7$). Unfortunately, the “noughties” (2000s) are a tricky time to be doing this activity, due to the abundance of zeros. The activities that follow get around this problem and offer some powerful advantages for teaching certain aspects of number.

Basic principles for “make this number” activities

In all of the “make this number” activities described below, the idea is to produce each of the numbers 1 to 20 as the answer in a number sentence constructed according to the given rules. As students become more experienced with number and the activities, or if you want to extend more able students, you can allow them to go beyond 20. Students may construct solutions on their own, or you may choose to allow some collaboration or work in teams. Students should keep an individual record of their solutions.

You can also create a large class chart with a grid containing the numbers 1 to 20 written in the corners of the cells. Students can then write up their own number expressions into the grid as they construct solutions. This allows multiple contributions and you can encourage students to check the work. You can also guide discussion of why multiple answers are possible, and why it is harder to construct expressions for some numbers.

In each of the versions of the activity, the “rules” are described, some example solutions are given, and some suggestions are made about how to conduct the particular activity in the classroom. Different activities lend themselves to bringing out certain mathematical principles; these are highlighted. In some cases, variations on an activity are suggested, but it is recommended that these are done after the initial activity has been explored.

2, 3, 5 (Version 1)

Suitable for: Year 2 and above
Rules: Use the digits 2, 3 and 5 as many or as few times as you like (or not at all), and whatever operations you want. You may choose to allow “concatenation,” so that, for example, you can write $18 = 23 - 5$, in which 2 and 3 are concatenated to form 23.
Examples:

$9 = 2 + 2 + 2 + 3$; $15 = 3 \times 5$; $13 = 3 \times 5 - 2$

Notes for teaching: With younger students it may be necessary to model how the rules work. Appropriate ways for recording the solutions should be modelled and reinforced. It may be useful to highlight problem-solving strategies, such as not having to complete the numbers 1 to 20 in order. Once students

have worked out a number, ask them if they can work out any nearby numbers by building on their results so far. Younger children are likely to focus on addition only, and so encouragement will be needed to help them look for ways of using subtraction although the activity itself will also force them to use it. Older children should be encouraged to incorporate multiplication.

Variations: (i) Can students find the shortest expression for each number (i.e., the one involving the least number of symbols)? (ii) What about the longest expression? Students should realise that they can add and take away the same thing as many times as they like, so that they could, for example, write

$$9 = 3 \times 3 + 2 - 2 + 2 - 2 + 2 - 2 + 2 - 2 + \ldots$$

This property—that adding and subtracting the same thing has no net effect—is a very powerful mathematical principle.

2, 3, 5 (Version 2)

Suitable for: Year 2 and above

Rules: Use the digits 2, 3 and 5 at least once each, and whatever operations you want. You may choose to allow "concatenation," so that, for example, you can write $18 = 23 - 5$.

Examples:

$$9=5+3+(2\div2); 15=3\times5+(2-2); 13=3\times5-2$$

Notes for teaching: This is more challenging than Version 1, because of the requirement to ensure that all three of the digits are used at least once each. With slightly older students, it can be powerful to conduct this activity immediately after Version 1 so that students can realise that they may have already obtained solutions in Version 1. In this situation you may also find some students who realise that they can take their answers from the previous activity, and do "clever things." So, for example, the number 5 might be obtained from $5 = 2 + 3 + 5 - 5$ or $5 = (2 + 3) \times 5 \div 5$ or even $5 = 5 + 2 - 2 + 3 - 3$. Recognising the power of adding "0" and multiplying by "1" is really important for students, as is recognising that 0 and 1 can be written in clever ways. It is useful to have a class discussion about these properties. Some students may feel that doing this is "cheating," but in fact it is powerful mathematics. If you have students who discover either trick, let them use it for a while so that its power is reinforced, congratulate them, and then challenge them to do the remaining problems using a different strategy.

Variations: (i) Can students find the shortest expression for each number (i.e., the one involving the least number of symbols)? (ii) What about the longest expression? See comments for Version 1. (iii) Can you find expressions that use one and only one of each number (e.g., $13 = 3 \times 5 - 2$). This is a hard challenge. There may be some numbers that cannot be expressed in this way using only the mathematical tools available at primary school level.

Adding

Suitable for: Year 4 and above

Rules: You are only permitted to use the plus sign, but you can use it as many times as you like. You can choose which numbers you are allowed to use, but the aim is to see who uses the fewest different numbers to produce the numbers 1 to 20.

Examples: Students might start with $9 = 5 + 4$, for example, and then realise that this means that they need to use a 5 and a 4. They might then recognise that they cannot make 3 with a 5 and 4 and so they need to include 3 in the set of needed numbers. Eventually they may realise that $9 = 2 + 2 + 2 + 2 + 1$ and $4 = 2 + 2$, and so maybe only twos and ones are needed. See notes for teaching for more discussion.

Notes for teaching: This is a very targeted

activity. It will require care in its conduct to ensure that students come to discover the very powerful result that it is intended to highlight, rather than having it told to them by the teacher or an enthusiastic student who blurts it out. The following approach might assist. After explaining the rules, allow students to try some possibilities. As you examine students’ work, let them know what numbers their suggestions have used, and encourage them to think about ways they might reduce the set of numbers they need. Before too long, students should come to realise that the only number that they need is 1: every number can be constructed by adding together a sufficient number of ones. This activity could be called “The Power of One,” but only after students have realised what is happening!

10-sided die (0 to 9)

Suitable for: Year 4 and above
Rules: Use a ten-sided die marked with the digits 0 to 9, and roll it four times. Use each of these four digits once only and any mathematical symbols.
Examples: Suppose the digits are 3, 5, 6, and 0. Then

$1 = 6 - 5 + (3 \times 0)$; $2 = (6 \div 3) + (5 \times 0)$;
$3 = 3 + (5 \times 6 \times 0)$; $4 = 6 - (5 - 3) + 0$; etc.

Notes for teaching: This version of the game will really force students to focus on order of operations and it will be important to emphasise the use of brackets to reduce ambiguity. The random nature of the selected digits will affect what particular number facts will be emphasised, but this will force students to think about an individual number in lots of different ways as they try to figure out how to “make” that particular number. Note that there may be some combinations of digits that will generate the numbers 1 to 20 easily, whereas others may prove difficult or even impossible for some numbers (e.g., I am struggling to think of a way to produce 17 out of 3, 5, 6, and 0 using only primary school mathematics).
Variations: If you find that some numbers are proving difficult and students have worked on them for a while, you may want to allow repeated use of digits (so, for example, we can now produce 17 as $17 = 3 \times 6 - (5 \div 5) + 0$, where I have had to use 5 twice).

Only fives

Suitable for: Year 5 and above
Rules: You may use as many fives as you like, but no other digits at all, and any mathematical symbols.
Examples:

$3 = (5 + 5 + 5) \div 5$ or $3 = 5 \div 5 + 5 \div 5 + 5 \div 5$

Notes for teaching: Initially students may strive to construct each number separately with no thought that there might be a general principle. Before they start, you may like to ask them whether or not they think they will be able to do all the numbers from 1 to 20; although this question is only worth asking if you have done other “make this number” activities recently. The limitation of only using fives may lead students to think that it is not possible. After some time working on this task, students should come to realise that all the numbers from 1 to 20 (and beyond) can be produced, either by adding together sufficient copies of $5 \div 5$ or by adding together the appropriate number of fives and then dividing by 5 (see the examples above). It is preferable if students can come to discover these strategies for themselves rather than being told by the teacher or some faster working student. This activity has the potential to help students realise why, for example, $\frac{7x}{x} = 7$. It can also reinforce the idea that we can write 1 as $x \div x$

or $\frac{x}{x}$ (provided x is not 0 of course!), and that every whole number can be made by adding together the required number of ones.

Variation: What is the least number of fives needed for each number? This activity should only be done after the important principles of the "as many fives" version described above have been learned. These general principles are unlikely to arise from the restricted "least number of fives" variation. Nevertheless, there is some interesting problem solving that can be done with this variation.

Multiplying

Suitable for: Year 6 and above

Rules: You are only permitted to use the multiplication sign, but you can use it as many times as you like. You can choose which numbers you are allowed to use, but the aim is to find the fewest different numbers required to make all the numbers we need. This will work better if you ask students to produce the numbers 1 to 100 instead of 1 to 20 only.

Examples: Students might realise that they can make 12 by doing 3×4. This means they need a 3 and a 4 in their "making set." In order to make 15, however, they will need a 5 as well (3×5). What else will they need? Do they really need a 4?

Notes for teaching: This is also a very targeted activity, which is like the "adding" version of "Make This Number." It too will require care when done in the classroom so that students have the chance to discover the results for themselves, rather than being told. A suggested approach is as follows. Let students work on the task for a short time, paying attention to what individual students discover. After the students have done a little exploration with the problem, have the whole class listen as you invite students to share what "making numbers" they have used, and what numbers they were able to make. So, for instance, the students who did the work in the example above have used 3, 4, and 5 to make 12 and 15, and of course they can make 20 as well. In this case, you can ask: "Can you make 14?", "Do you need a 4?", and "What will you need to make 13?". Do not give too much away with these questions. Allow the students to continue to explore and refine what numbers they absolutely need in their "making set," encouraging them to think about ways they might reduce the set of numbers they need. Eventually they should come to discover that the "making set" has to include 1, 2, 3, 5, 7, 11, 13, 17, and so on; in other words, the prime numbers together with 1. This is because the prime numbers are the building blocks for numbers using multiplication. If any students discover the result quickly, give plenty of praise but encourage them to keep the result to themselves.

Four fours

Suitable for: Year 6 and above

Rules: You must use the digit 4 exactly four times, and you may use any mathematical operations.

Examples:

$$11 = \frac{44}{\sqrt{4} \times \sqrt{4}}$$

Notes for teaching: This is a challenging task for Year 6 students, and so it may be appropriate to do this in small groups, or even as a whole class challenge - and conducted it as an extended activity over days or even weeks. Students usually start off using just the four basic operations (addition, subtraction, multiplication and division), but may find that these are insufficient for producing the numbers 1 to 20. You may need to remind them of the importance of using brackets to reinforce the necessary order of operations. Ideally some student will point out that it is permissible to use $\sqrt{4}$, although of course this uses up one of the fours. Finally, there is one

number between 1 and 20 that, to the best of my knowledge, needs one extra mathematical tool that most primary school students will not know. The idea of “factorial” is not hard, and can be explained easily. The number 6 factorial, for example, is written as 6! and is just the product of all the whole numbers from 6 down to 1, i.e., $6 \times 5 \times 4 \times 3 \times 2 \times 1$, which is 720. For the present activity, it is 4! that is useful. This single symbol 4!, using just one 4, gives us $4 \times 3 \times 2 \times 1$ or 24. You may choose when to “release” this information to students, because it can be useful to allow students to wrestle with a lack of success for a while. (Factorials are used a lot in the mathematics of counting arrangements of things, such as how many different ways four people can sit on four chairs.) These basic and semi-advanced tools will get students from 1 to 20 and some way beyond as well.

Note: I first met the four fours problem as a Year 9 student longer ago than I am going to confess. Our class was challenged to find the numbers 1 to 100, and in the years that followed I managed to find all but four numbers, although I had to use some post-primary school mathematics for some of them, including $\cdot\bar{4}$(i.e., 0.4444…). I have since found additional material on the problem, including an explanation for how to generate every positive integer (Stewart & Jaworski, 1981; fortunately this reference is hard to come by because I do not want to spoil your fun!).

Conclusion

The “make this number” activities presented here will provide your students with plenty of opportunities to make numbers their friends. These activities offer a challenge without being overly competitive or forcing students to work fast or under unpleasant pressure. It does not matter if someone comes up with a different solution five minutes or a week after the first solution is discovered. Some versions of “Make This Number” could be conducted frequently: the 10-sided die activity, for example, could be run once a week, since different sets of numbers will be generated each time. Through the use of these activities students will gain fluency with basic number facts, appreciate order of operations and the use of brackets, build an understanding of key number properties, gain an appreciation of 1 and the prime numbers as building blocks, and develop problem solving skills. They are additional tools that can be added to the teachers’ repertoire of activities that help students develop a deeper knowledge of numbers, relationships and properties.

References

McIntosh, A., DeNardi, E. & Swan, P. (1994). *Think mathematically: How to teach mental maths in the primary classroom.* Melbourne: Longman Cheshire.

Stewart, I. & Jaworski, J. (Eds) (1981). *Seven years of ‘Manifold’, 1968–1980.* Cheshire, UK: Shiva Publishing Ltd.

Helen Chick
The University of Melbourne
<h.chick@unimelb.edu.au>

APMC